Achievements & Challenges in the Mathematics of Complexity Science

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Outline

1. Selected achievements
2. Selected challenges
3. Conclusion

1. Selected achievements

- Brin & Page
- Kesten & Sidoravicius
- Guerra & Talagrand
- Gielis & MacKay

Brin & Page

- Google's search engine (1998)
- Scoring webpages for significance via weight for a random walk on the web with random reinjection

Kesten & Sidoravicius

Angels & Mortals, S.Solomon et al, 2000:

Suppose a 2D lattice populated by As and Bs, and each time step a probability of:

As and Bs moving to neighbouring sites;

B -> 0 (“death”);

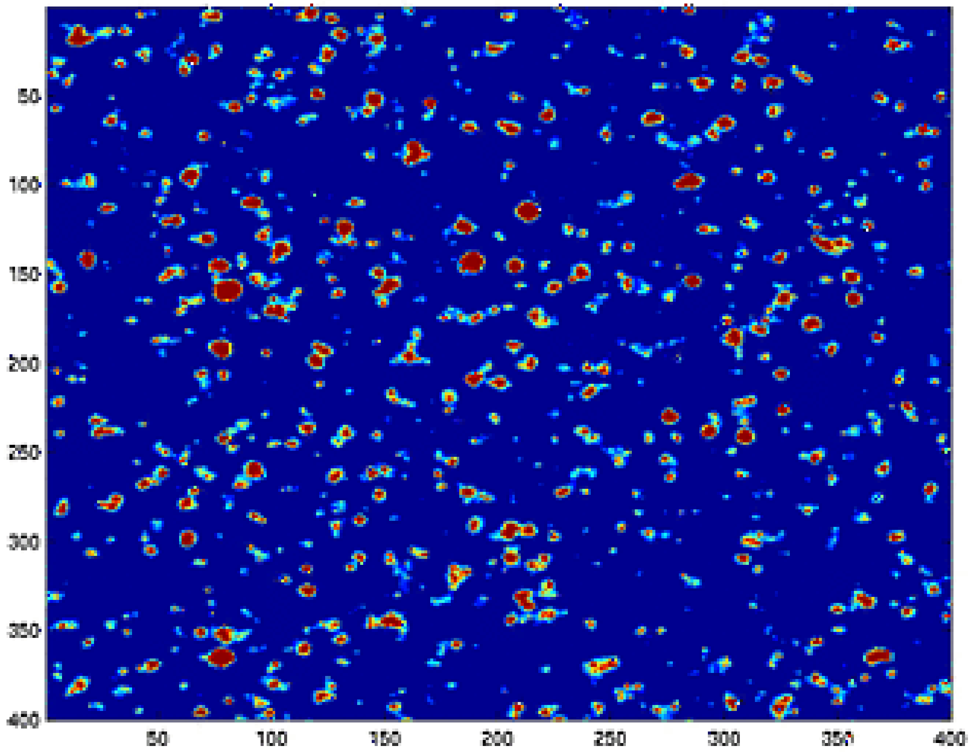
A+B -> A+2B (“catalysis”).

Ecological and economic interpretations.

If death rate > catalysis rate * density of A, expect Bs to die out?

Eldad Bettleheim

No!



Enter the mathematicians

Let NB = number of Bs at a given site.

Kesten and Sidoravicius (2003) proved:

- if death rate exceeds a function of catalysis rate, then NB = 0 for all large enough time;
- yet whatever the parameter values, the expectation of NB goes to infinity.

So in the large death rate regime, get long-lasting clumps of large activity in a growing sea of inactivity.

Guerra & Talagrand

- Sherrington-Kirkpatrick spin glass model $H(s) = h\sum s_i - \beta\sum g_{ij}s_is_j$,
  $s_i = \pm 1$, $i = 1..N$, g_{ij} iid Gaussian(0,1/N)
- Parisi's result via replica trick
  $\langle 1/N \log \sum_s \exp H(s)\rangle \to \inf_k P_k$,
  an expression based on k "replicas"
- Result (but not method) proved by Talagrand (2003).

Gielis & MacKay

- Bunimovich & Sinai conjecture (1988): there are indecomposable coupled map lattices with non-unique space-time phase (statistical behaviour compatible with the dynamics).
- Proved by Gielis & MacKay, 2000 (cf. Yamaguchi)

Sorts out Emergence

- What emerges from a complex system is a space-time phase
- Amount of emergence = distance of space-time phase from a product
- Strong emergence = non-unique space-time phase
- Amount of strong emergence = diameter of set of space-time phases
- MacKay, Nonlinearity 21 (2008) T273

2. Selected challenges

- Answers for agent-based models
- Aggregation procedures
- Theory of space-time phases
- Mathematical formulation of socially relevant problems
- Data analysis to test and use models

Answers for agent-based models (ABM)

- ABMs are touted as THE way to do social science (e.g. J Epstein).
- Most proponents would like more than just the output of some simulations,
- but they don’t seem to realise there is a huge body of mathematical results on ABM, under the name “Interacting Particle Systems” (e.g. Liggett’s 2 books) (admittedly often for idealised classes of system).

For example

- Epstein finds exponentially long time to absorption with respect to number of agents and size of memory,
- but exponentially long time to absorption is proved for many classes of system with an absorbing state, e.g. contact processes above the epidemic threshold.

For example

- Sawyer postulates that “near decom-posability implies little emergence”,
- but this has a rigorous basis in “weak dependence implies unique space-time phase and it is close to a product” (based on ideas of Dobrushin).

In contrast

- Sawyer supports the “supervenience principle” that a change in macro-level properties can not arise without a change in micro-level properties,
- but this is false because spatially extended systems can have more than one space-time phase.

Need to make sense of

- Downward causation
- Distinction between cognitive (or adaptive) and reactive agents

Aggregation procedures

- Want to deduce macro from micro
- One way is successive aggregation: prove that the macro effects of a model are unchanged if one aggregates groups of agents into "super-agents" with suitable attributes, and iterate the procedure.
- Two contexts where I think this can be achieved rigorously:

- Selfish traffic flow with junction costs
- Synchronisation in networks of oscillators

Space-time phases

- Correlations, statistics… (eg as Epstein wants for his civil violence model)
- Size of set of phases
- Parameter-dependence
- Control
- Design

Mathematical formulation of socially relevant problems

- Includes ABM, e.g. Epstein does well
- Formulate model for “asabiya” (capa-city for collective action, Ibn Khaldun, P Turchin)
- Healthcare delivery
- Sustainable communities
- Demographics
- Emergence of norms, beliefs

Data analysis

- E.g. Colleagues in healthcare come to us with massive datasets and want to know what to do with them.
- Seems to me mainly fitting multivariate probability distributions, and then formulating decisions.

Conclusion

- “Appliquons aux sciences politiques et morales la méthode fondée sur l’observation et sur le calcul, méthode qui nous a si bien servi dans les sciences naturelles” (Laplace, Essai philosophique sur les probabilités,1814)
- Preceded by Hobbes, followed by Comte, Durkheim…